Math Review

$\mathbb{N}$: natural numbers
$\{0, 1, 2 \ldots\}$

$\mathbb{Z}$: integers
$\{-1, 1, -2, 2, 0 \ldots\}$

$\mathbb{Z}^+$: positive integers
$\{1, 2, \ldots\}$

$\mathbb{R}$: real numbers
$\{-2, 2.5, \pi\}$

$\mathbb{Q}$: rational numbers
m/n , m and n are integers

$\mathbb{C}$: $\{i, 2i+1, \pi\}$

$x \in \mathbb{R}$
↑
set element

$y \in (0, 5]$ $y \in \mathbb{Z}$
$\{1, 2, 3, 4, 5\}$

(a, b) where $a \in \mathbb{R}, b \in \mathbb{R}$
$(a, b) \in \mathbb{R}^2$ ~~squared numbers~~

exponents and logs

b^n , $b^0 = 1$, $b^{.5}$ or $b^{1/2} = \sqrt{b}$, $b^{-1} = 1/b$

rules: $b^x b^y = b^{x+y}$
$b^x a^x = (ba)^x$
$(b^x)^y = b^{(xy)}$

$y = b^x \longleftrightarrow x = \log_b y$ if $b > 1$, $y > 0$
if b not given, assume $b = 2$

$b^{\log_b(x)} = x$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b(x^y) = y \cdot \log_b x$

$\log_b x = \log_a x \log_b a$

$\log_a x = \dfrac{\log_b x}{\log_b a}$

$k! = 1 \cdot 2 \cdot \ldots \cdot k$

$\max(x, y)$ = bigger of x & y

$\lfloor 3.7 \rfloor = 3$ $\lceil 3.7 \rceil = 4$

$\lfloor -2.1 \rfloor = -3$ $\lceil -2.1 \rceil = -2$

Logic

propositional logic

proposition : a statement which is either true or false

ex) 5 is odd

we are in Siebel right now

complex proposition : $\underbrace{\text{Naina is from PA}}_{\text{T or F}}$ $\boxed{\text{and}}$ $\underbrace{\text{Charlotte is from NY}}_{\text{T or F}}$

$\underbrace{\text{T or F}}_{p}$ $\underbrace{\text{T or F}}_{q}$

and : $\wedge$

or : $\vee$

implies : $\longrightarrow$

not : $\neg$ (on a single proposition)

truth tables = definition

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

inclusive

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

vacuous

if $\underbrace{\text{it is raining}}_{p}$, then $\underbrace{\text{I will bring an umbrella}}_{q}$.

p	$\neg p$
T	F
F	T

logical equivalence

two statements are logically equivalent if they evaluate to True / False under the same conditions.

De Morgan's Laws:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T